Problem set 8

Due May 9, 2024

1. Starting from the expression of the Boltzmann-averaged energy of an ideal monoatomic gas composed of N point-like particles

$$E = \frac{3}{2} N k_B T$$

derive the formula for its heat capacity at constant volume (C_V).

2. How can be the energy equipartition principle seen from the obtained expression for C_V?

3. Based on the expression for C_V derive the expression for the standard deviation of the average energy (σ_E) for an ideal monoatomic gas and for the relative standard deviation of energy (σ_E/E). Compute σ_E/E for 1 picomol (10^{-12} mol) of an ideal gas and comment on the obtained value.

   The relationship between energy variance (σ_E^2) and C_V is

$$\sigma_E^2 = k_B T C_V$$

4. Based on the equation of state of an ideal gas (the Clapeyron equation) and the relatioship between the variance of density and isothermal compressibility coefficient (κ)

$$\kappa = -\frac{1}{V}\frac{\partial V}{\partial p} = \frac{V}{k_B T \rho^2}\sigma_\rho^2$$

with

$$\rho = \frac{N}{V}$$

where N is the number of atoms, V is the volume and p is the pressure, derive the expression for the relative standard deviation of density, σ_ρ/ρ and show that this quantity is independent of pressure. Calculate σ_ρ/ρ for for 1 picomol of an ideal gas and comment on the obtained value.